16.3 More about algebra

Signs and symbols

If you used symbols in your GCSE course, you might have met the use of s for distance and I for current. Maybe you wondered why we don't use d for distance instead of s or C for current instead of I. The answer is that physics discoveries have taken place in many countries. The first person to discover the key ideas about speed was Galileo, the great Italian scientist, so he used the words 'scale' from his own language for distance and therefore assigned the symbol s to distance. Important discoveries about electricity were made by Ampère, the great French scientist, and he wrote about the intensity of an electric current, so he used the symbol I for electric current. The symbols we now use are used in all countries in association with the **SI system of units**.

Table 1 *Symbols for some physical quantities*

physical quantity	symbol	unit	unit symbol
distance	s	metre	m
speed or velocity	v	metre per second	$\mathrm{m\,s^{-1}}$
acceleration	a	metre per second per second	$\mathrm{m\,s^{-2}}$
mass	m	kilogram	kg
force	F	newton	N
energy or work	E	joule	J
power	P	watt	W
density	ρ	kilogram per cubic metre	$\mathrm{kg\,m^{-3}}$
current	I	ampere	A
potential difference or voltage	V	volt	V
resistance	R	ohm	Ω

Signs you need to recognise

- Inequality signs are often used in physics. You need to be able to recognise the meaning of the signs in Table 2. For example, the inequality $I \geq 3\,\mathrm{A}$ means that the current is greater or equal to 3 A. This is the same as saying that the current is not less than 3 A.
- The approximation sign is used where an estimate or an order-of-magnitude calculation is made, rather than a precise calculation. For an order-of-magnitude calculation, the final value is written with one significant figure only, or even rounded up or down to the nearest power of ten. Order-of-magnitude calculations are useful as a quick check after using a calculator. For example, if you are asked to calculate the density of a 1.0 kg metal cylinder of height 0.100 m and diameter 0.071 m, you ought to obtain a value of $2530\,\mathrm{kg\,m^{-3}}$ using a calculator. Now let's check the value quickly:

  $$\text{Volume} = \pi(\text{radius})^2 \times \text{height} = 3 \times (0.04)^2 \times 0.1 = 48 \times 10^{-5}\,\mathrm{m^3}$$

  $$\text{Density} = \text{mass/volume} = 1.0/50 \times 10^{-5} = 2000\,\mathrm{kg\,m^{-3}}$$

  This confirms our 'precise' calculation.
- Proportionality is represented by the $\propto$ sign. A simple example of its use in physics is for Hooke's law; the tension in a spring is directly proportional to its extension.

$$\text{Tension } T \propto \text{extension } \Delta L$$

By introducing a constant of proportionality k, the link above can be made into an equation:

$$T = k\Delta L$$

where k is defined as the spring constant. See Topic 11.2. With any proportionality relationship, if one of the variables is increased by a given factor (e.g. × 3), the other variable is increased by the same factor. So in the above example, if T is trebled, then extension ΔL is also trebled. A graph of tension T on the y-axis against extension ΔL on the x-axis would give a straight line through the origin.

Table 2 *Signs*

sign	meaning	sign	meaning	sign	meaning
$>$	greater than	$>>$	much greater than	$<x^2>$	mean square value
$<$	less than	$<<$	much less than	$\propto$	is proportional to
$\geq$	greater than or equal to	$\approx$	approximately equals	Δ	change of
$\leq$	less than or equal to	$<x>$	mean value	$\surd$	square root

More about equations and formulae

Rearranging an equation with several terms

The equation $v = u + at$ is an example of an equation with two terms on the right-hand side. These terms are u and at. To make t the subject of the equation,

1 Isolate the term containing t on one side by subtracting u from both sides to give $v - u = at$

2 Isolate t by dividing both sides of the equation $v - u = at$ by a to give

$$\frac{(v-u)}{a} = \frac{at}{a} = t$$

Note that a cancels out in the expression $\frac{at}{a}$

3 The rearranged equation may now be written $t = \frac{(v-u)}{a}$

Rearranging an equation containing powers

Suppose a quantity is raised to a power in a term in an equation, and that quantity is to be made the subject of the equation. For example, consider the equation $V = \frac{4}{3}\pi r^3$ where r is to be made the subject of the equation.

1 Isolate r^3 from the other factors in the equation, by dividing both sides by 4π, then multiplying both sides by 3 to give $\frac{3V}{4\pi} = r^3$

2 Take the cube root of both sides to give $\left(\frac{3V}{4\pi}\right)^{1/3} = r$

3 Rewrite the equation with r on the left-hand side if necessary.

More about powers

1 Powers add for identical quantities when two terms are multiplied together. For example, if $y = ax^n$ and $z = bx^m$, then $yz = ax^m bx^n = abx^{m+n}$

2 An equation of the form $y = \frac{k}{z^n}$ may be written in the form $y = kz^{-n}$.

3 The n^{th} root of an expression is written as the power $1/n$. For example, the square root of x is $x^{1/2}$. Therefore, rearranging $y = x^n$ to make x the subject gives $x = y^{1/n}$.

Summary questions

1 Complete each of the following statements:

a If $x > 5$, then $1/x <$

b If $4 < x < 10$, then _____ $< 1/x <$

c If x is positive and $x^2 > 100$ then $1/x$ _____.

2 a Make t the subject of each of the following equations:

i $v = u + at$, ii $s = \frac{1}{2}at^2$, iii $y = k(t - t_o)$, iv $F = \frac{mv}{t}$

b Solve each of the following equations:

i $2z + 6 = 10$, ii $2(z + 6) = 10$, iii $\frac{2}{z-4} = 8$,

iv $\frac{4}{z^2} = 36$

3 a Make x the subject of each of the following equations:

i $y = 2x^{1/2}$, ii $2y = x^{-1/2}$, iii $yx^{1/3} = 1$, iv $y = \frac{k}{x^2}$

b Solve each of the following equations:

i $x^{-1/2} = 2$, ii $3x^2 = 24$, iii $\frac{8}{x^2} = 32$, iv $2(x^{1/2} + 4) = 12$

4 Use the data given with each equation below to calculate:

a the volume V of a wire of radius $r = 0.34$ mm and length $L = 0.840$ m, using the equation $V = \pi r^2 L$,

b the radius r of a sphere of volume $V = 1.00 \times 10^{-6}\,m^3$, using the formula $V = \frac{4}{3}\pi r^3$,

c the time period T of a simple pendulum of length $L = 1.50$ m, using the formula $T = 2\pi(L/g)^{0.5}$, where $g = 9.8\,m\,s^{-2}$,

d the speed v of an object of mass $m = 0.20$ kg and kinetic energy $E_k = 28$ J, using the formula $E_k = \frac{1}{2}mv^2$.